The Gyroscope

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Gyroscopes behave in very strange ways. Many demonstrations are given with little explanation just to keep the audience guessing. When you search for the answers, you are told that it is the angular momentum, the right hand rule and you are shown some equations. But the explanation is not very clear. This paper suggests that angular momentum is not the best explanation. Rather, linear velocity, cohesion, and gravity are all that is needed to understand the strange movements of the gyroscope.

1. Gyroscopic Motion

Take a toy gyroscope, set it on a pedestal and it falls down. Oops! I forgot to spin it. OK, take a toy gyroscope, spin it, and set it on a pedestal. It does not fall. It moves around and around and then after a short period of time, it falls.

What keeps it up?
What causes it to move in a circle?
Is it weightless??

It seems quite clear that this strange motion of gyroscope is caused by the spinning of the rotor.

2. The Normal Explanation

When you read reports that explain the gyroscope, here is what you find:

Angular momentum
Right hand rule
Math

Using these ideas, showing drawings of the gyroscope, and showing the force vectors; we are expected to understand how it works.

But it does not help!

Is it the Momentum?

There are two types of momentum, linear and angular. We have all experienced both of them. When you were young, were you hit by a baseball? Did it hurt? Of course it did. The faster the ball flies or a ball with a greater mass, it will hurt more because it has more momentum. That's linear momentum.

Have you ever tried to stop a spinning bicycle tire? The tire will push your hand in the direction of its spin. Depending on the mass of the tire and its angular velocity, the angular momentum can be very strong. It will almost tear your fingers off.

Unfortunately, the common explanation states that the direction of angular momentum **L** is determines by the right hand rule.

Is it the Right Hand Rule?

Figure 1 is an example of the right hand rule. The fingers of the right hand are curved in the same direction as the spin of the rotor. The thumb determines the direction of the angular momentum. This direction of the angular momentum is perpendicular to the spin of the rotor, while the direction of linear momentum is in line with the linear velocity. Can you explain that?

Is the **ω** by the thumb is wrong? Should it be **L** instead?

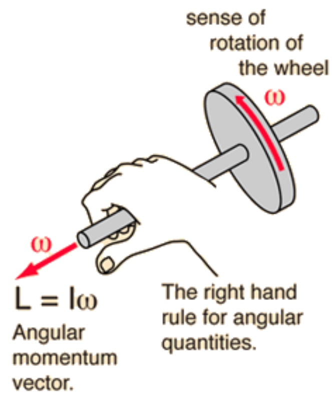

Figure 1 – The Right hand Rule

If the spinning tire pushes your hand in the direction of the spin, why is the angular momentum of the gyroscope described as being perpendicular to the spin?

Is there a feature of the gyroscope that requires this to be true?

Is it the Math?

$$\vec{L} = I\vec{\omega}. \quad (1)$$

Equation 1 is the equation for angular momentum. It indicates that the angular momentum **L** is a vector that should be in the same direction as the angular velocity **ω**.

$$I = {}^{2}/_{5}\, mR^2 \quad (2)$$

Equation 2 is the moment of inertia of a solid sphere. It is generally understood to be equivalent to linear mass as used in the equation for linear momentum. I is stated in units of Kg m/s.

$$k = \sqrt{I/m} \quad (3)$$

Equation 3 is the radius of gyration. Concentrate all of the mass of the rotor in a particle. 'k' is the distance that the total mass of the rotor is from the axle. With the mass in this position and spinning at an angular velocity of ω, the value of angular momentum is maintained.

Figure 2 shows a typical position where the mass could be concentrated and is moving at the angular velocity of ω. Clearly the effect of angular momentum shown in Figure 2 has the same effect as the spinning tire.

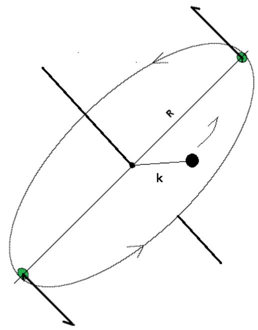

Figure 2 – Mass Concentrated at Distance k

Caused by Cross Products?

Torque is normally explained using the cross product of a force vector and a distance vector. It turns out that the resulting vector has a magnitude and direction. For some reason the resultant vector is defined as being perpendicular to the force and distance vectors. Because it is the only choice left? But to emphasize the confusion, the resultant vector is called a pseudo-vector. It's not real!

I have difficulty with the idea that distance is a vector. So the resultant torque vector is a product of force times the distance, and the direction is in the same direction as the force vector, that works for me! So the angular momentum vector is in the same direction as the angular velocity vector, which is what we experience with a spinning tire.

3. More about Angular Momentum

It is an Effect

Angular momentum is not a cause, it is an effect. It is a combination of linear momentum and cohesion. So, what is the cause of linear momentum? Even Newton did not answer this question. But he did state the idea in his first principle of motion. He called it inertia. I prefer to call it linear momentum or linear velocity. In any case it happens and we just have to accept it until we find a better answer.

Cohesion is my word that combines all the forces that hold particles together. They are nuclear forces, electromagnetic forces, and even gravity. It is this force that keeps a given particle at a given distance from the center of rotation.

There is no outward component

One effect of spinning is the existence of the equatorial bulge. No matter how you look at angular momentum, there is no way that it can produce an outward radial component. It moves in a circle.

Linear momentum has a radial outward component and as such can explain the equatorial bulge.

4. Stability and Position

The chart below compares the values of momentum and velocity using a lead ball and the earth. The lead ball has a radius of 0.1 meters and is spinning at the rate of 6,000 RPM. The earth has a radius of 6,371,000 meters and spins at 1 revolution per day.

	Lead Ball	**Earth**
Angular Momentum	1.19E+2	7.05E+33
Linear Momentum	2.98	2.76E+24
Angular Velocity	6.28	7.27E-7
Linear Velocity	62	463

Chart 1 - Comparison

When observing the motion of a toy gyroscope, it is clear that it does not take very long for the gyroscope to fall. On the other hand, the earth has been spinning and orbiting for a very long time and has not fallen into the sun. The momentum of the earth is much greater than the momentum of the lead ball. It seems that momentum speaks to stability.

However when calculating the next position of the rotor, velocity is used. So, for the rest of the paper it is not the angular or linear momentum that is used, it is the linear velocity.

5. The Next Position

Using an ideal gyroscope (the mass is perfectly balanced) with linear velocity and cohesion, a gyroscope will maintain its position. See Figure 3.

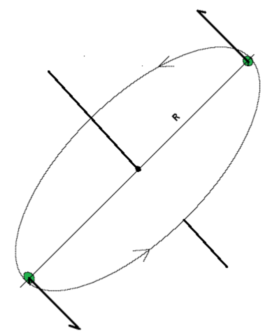

Figure 3 – Linear Velocity and Cohesion

There are two small pieces of the rotor shown in Figure 3. Each one has a linear velocity as shown. If they were free particles, they would fly out in the direction shown. Of course they are not free because they are held in by cohesion

So, we can use the model to predict where the two pieces will be in a small increment of time. It is not only the direction of the linear velocity nor the direction of the cohesive force. It is their magnitude. The linear velocity of the two small pieces is shown in Chart 1. But it is the cohesion that is the strongest. It is so strong that the radius R of the rotor shows no visible signs of change.

To calculate the next position, assume the small sphere is free and would move a distance Δx, Δy, and Δz in the time of Δt. Then use cohesion to move it back to a fixed distance R. With no other forces present, each small piece will move in a circle around the axle.

It will continue to do this as long as there is no imbalance of mass or forces.

6. Perfect Balance

Figure 4 shows a perfectly balanced gyroscope.

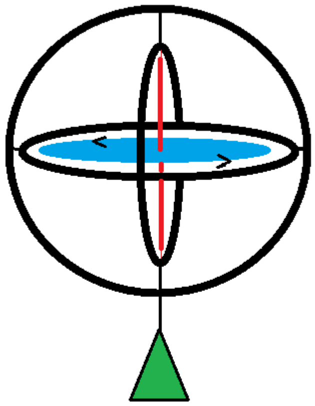

Figure 4 – Perfectly Balanced

If the mass of the gyroscope, including rotor and gimbals, is perfectly balanced; and the axle is perfectly lined up with gravity; and there are no other forces, the gyroscope will hold its position for ever. But the real world is not so kind. Any unbalance of mass or alignment or external forces will cause the gyroscope to tilt and eventually fall.

7. Falling Over

So let's assume you have a spinning gyroscope, you set it on a pedestal, and the axle is perfectly aligned with gravity. If there is a very small imbalance of mass, gravity will act on the imbalance and will tilt the axle a small amount in the direction of the extra mass. The amount of tilt that the axis assumes depends on the amount of the imbalance of mass. This explains why the gyroscope initially drops to a small or larger angle. This type of imbalance will cause the gyroscope to wobble; not to precess.

It turns out that while the gyroscope is precessing it continues to fall or to have a greater tilt. This happens as the linear velocity of the rotor slows down and force of gravity becomes more dominate.

8. Precession

Once the axle is tilted, the gyroscope will precess. Precession is the circular movement of the axle. We observe the movement of the axle, but it is the rotor that is in control. Further, the precession appears to be consistent in direction and velocity.

Direction

If you spin the rotor clockwise (CW), the axle moves CW. If you spin the rotor Counter-clockwise (CCW), the axle moves CCW. It happens every time. The direction of spin is always viewed from above the gyroscope and in the direction of the force of gravity.

We say the earth spins CCW because we observe the spin from the North Pole. But the reality is that the North Pole is established when we observe the CCW spin. So the same convention is used for the gyroscope.

The direction of spin is deterministic because the environment on your desk top has almost no extraneous forces that could change the motion. There is not much wind, nor magnetic and electrostatic fields, nor mechanical forces. But there is gravity. And for the direction to be consistent, the imbalance of gravity must be in the same direction all the time. Figure 5 shows a tilted gyroscope spinning CCW with the force of gravity applied to two pieces of the rotor.

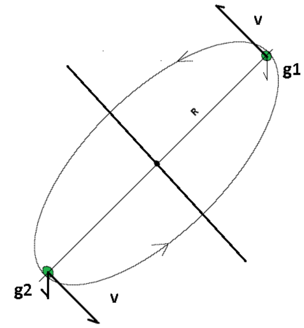

Figure 5 – The imbalance of Gravity

In the figure, the gravitational acceleration for the lower piece (g2) is greater than the gravitational acceleration of the upper piece (g1). Although both pieces are being pushed down the there is a net torque on the rotor at position g2. The common explanation of precession uses torque but it is applied at the center of the rotor and no further explanation is given.

The purpose of putting the net torque on g2 is to calculate the next position of the lowest point of the rotor. The lowest point of the rotor is marked by the position of the top of the axle. By calculating its next position, the movement of the rotor can be determined. The rotation of the axle will follow.

This lowest point moves slowly around in one direction. All the points below the horizon have a net force pushing them down. So as the linear velocity moves the points CCW, and the cohesion holds the points to a fixed distance from the axle, gravity pushes the points down. And so the lowest point on the rotor moves CCW.

Although this explanation seems reasonable, I am not absolutely convinced. The calculations need to be done.

Velocity

There is a video of Professor Laithwaite demonstrating the mysteries of the gyroscope to a young group of students in 1974. Reference [1] provides the link to this video. In the 4th part of the video, he demonstrates that the precession rate is determined by the amount of torque applied. There is a derivation of the velocity of the precession provided by Georgia State University [2]. The final result is shown in Equation 4.

$$\omega_p = mgd/I\omega_s \tag{4}$$

ωp is the angular velocity of precession
m is the total mass of the gyroscope
d is the distance
from the pivot point to the center of mass
I is the moment of inertia
ω_s is the angular velocity of the rotor

The torque in the equation is mgd, so it is clear that if the torque is doubled, the velocity is doubled. But I do not agree with how it was done. It starts with the angular momentum being directed along the axis of the gyroscope. If it starts with this, I can't proceed. I must go back to Figure 5 as my basic model and develop it further. It would get more complicated and be difficult to calculate, but it would be based on what I consider to be valid physics principles.

However, equation 4 may still be correct since I have not run a test or performed any calculations.

9. Calculating the Direction and Velocity

Figure 6 is my model for calculating the direction and velocity of a spinning gyroscope.

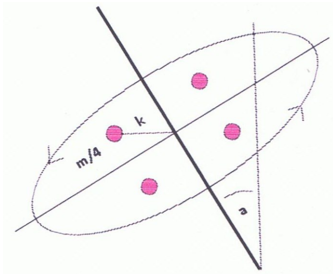

Figure 6 – Model for Calculation

The mass of the rotor is broken into four equal parts and placed at equal intervals around the axis each at a distance of k The angular velocity is **ω** and the angle of inclination is **a.** The pivot point of the axle is at sea level and is the origin of the c0-ordinate system. The forces of gravity are applied at each of the four points. As a starting point for the calculation, I will use this model and the following algorithm.

The lowest point of the rotor is in the same direction as the direction of the axle.

Calculate the next position of the rotor for at least one revolution or more.

At the end, find the lowest point of the rotor.

The rate of precession is the change in angle of the lowest point divided by the elapsed time.

The direction of the precession is determined by the sign of the change in angle.

Compare the result with the GSU equation.

The GSU equation does not emphasize the direction.

10. Summary

Weightless

Clearly the spinning gyroscope appears to float. But is it weightless? That is, does it have zero weight? Equation 4 has the term 'mg' that represents the force on the center of the gyroscope and 'm' is the total mass of the gyroscope. The equation is stating that all of the mass is being acted upon by gravity. So the mass is not less and gravity is not less. So it is not weightless!

It floats because the linear velocity is greater than the force of gravity. So the rotor tries to stay where it is. As the rotor slows down the linear velocity decreases and gravity has a greater effect causing the gyroscope slowly fall over.

Angular Momentum

Angular momentum is used to explain the strange movements of the gyroscope. However, it is called a pseudo-vector because it is not real, which makes the right hand rule null and void. Angular momentum does speak to the issue of stability. It is linear velocity that determines the next position of the rotor and provides the best explanation of the gyroscopes strange behavior.

Precession

Precession does seem deterministic in both direction and speed.

References

[1] – Professor Eric Laithewaite 1974 Demonstrations
http://www.intalek.com/Index/Projects/Research/Laithwaite/Laithwaite1974.htm

[2] – Georgia State Universities equation for angular momentum
http://hyperphysics.phy-astr.gsu.edu/hbase/top.html